Determination of local elastic strains by EBSD techniques

S. Villert [a,b], C. Maurice [a], C. Wyon [b,c], R. Fortunier [a*]

[a] Centre SMS, Ecole des Mines, 158 cours Fauriel, 42023 Saint-Etienne cedex 02, France
[b] CEA-DRT-LETI, 17 Avenue des Martyrs, 38054 Grenoble cedex 9, France
[c] ST Microelectronics, 860 rue Jean Monnet, 38920 - CROLLES

Résumé – Local elastic strains can be derived from EBSD diagrams by using two approaches based on image processing and minimisation procedures. The "pattern shift" method is based on the cross-correlation of two diagrams, whereas the "3D Hough" method gives an accurate analysis of a unique diagram. Both methods are briefly described.

1. Introduction

Electron Back Scattering Diffraction (EBSD) is now commonly used to analyse local crystallographic orientations in metals and alloys. The spatial resolution is the main advantage of this method. Crystallographic texture maps can be drawn with a spatial resolution better than 50 nanometres. Since this technique is basically derived from the Bragg law, it became natural to apply it to the determination of local elastic strains. This has been done first by Troost [1], who has developed the "pattern shift" method. According to this method, relative elastic strains and spin can be derived by correlating two EBSD diagrams [2, 3]. More recently, a new method has been developed to determine the absolute values of the elastic strains from a unique EBSD diagram. This method is now known as the "3D Hough" method [4]. This paper gives recent results obtained by both methods.

2. « Pattern shift » method

The pattern shift method has been first investigated by Troost [1]. It has then been developed and applied to experimental EBSD diagrams by Wilkinson [2, 3]. The main principles of this method are the following:

- Two EBSD diagrams are compared by cross-correlation techniques. The homogeneous displacements Δx_i and Δx_i of M regions of interest or zone axes are determined (i=1… M).
- The homogeneous displacement δx_i and δy_i of the same regions of interest or zone axes are calculated as a function of the displacement gradient tensor **d**.
- The following cost function F is minimised in order to obtain the displacement gradient tensor **d**:

$$F(\mathbf{d}) = \frac{1}{2}\sum_{i=1}^{M}\left[\left(\delta x_i(\mathbf{d}) - \Delta x_i\right)^2 + \left(\delta y_i(\mathbf{d}) - \Delta x_i\right)^2\right] \quad (1)$$

It should be noted that, by using this method, only eight of the nine components of the **d** tensor can be determined. In fact, the displacements obtained by cross-correlation do not depend on the trace of this tensor, i.e. on the volume change due to elastic strain. It should be also noted that at least four zone axis or region of interest have to be analysed, since we need at least eight experimental results to obtain a well posed problem.

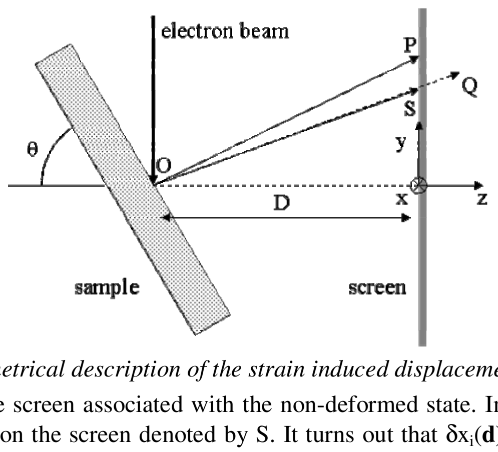

Figure 1 – Geometrical description of the strain induced displacements in the screen.

In figure 1, P is a point on the screen associated with the non-deformed state. In the deformed state, this point becomes Q, with a projection on the screen denoted by S. It turns out that $\delta x_i(\mathbf{d})$ and $\delta y_i(\mathbf{d})$ in equation (1) are respectively the x and y components of the **OS**-**OP**=(D/OQ_z)**OQ**-**OP**. Now, if **R** denotes the rotation matrix associated with the tilt angle θ, and **I** the identity, then we have **OQ**=**R**.(**I**+**d**).$\mathbf{R}^t$.**OP**. It turns out that $\delta x_i(\mathbf{d})$ and

* Auteur à contacter : fortunier@emse.fr – Tel : 04 77 42 00 34

$\delta y_i(\mathbf{d})$ are non-linear functions of **d**. However, by multiplying the cost function in equation (1) by OQ_z, a new cost function F' can be defined as follows:

$$F'(\mathbf{d}) = \frac{1}{2}\sum_{i=1}^{M}\left[\begin{array}{l}\left(D\left(\mathbf{R}.\left(\mathbf{I}+\mathbf{d}\right).\mathbf{R}^t.\mathbf{OP}\right)_x - \Delta x_i\left(\mathbf{R}.\left(\mathbf{I}+\mathbf{d}\right).\mathbf{R}^t.\mathbf{OP}\right)_z\right)^2 \\ +\left(D\left(\mathbf{R}.\left(\mathbf{I}+\mathbf{d}\right).\mathbf{R}^t.\mathbf{OP}\right)_y - \Delta y_i\left(\mathbf{R}.\left(\mathbf{I}+\mathbf{d}\right).\mathbf{R}^t.\mathbf{OP}\right)_z\right)^2\end{array}\right] = \frac{1}{2}\left\|\mathbf{A}.\mathbf{u}-\mathbf{b}\right\|^2 \quad (3)$$

In this equation, **u** is an 8 components vector containing the d_{ij} values; **A** is a 2M×8 matrix and **b** a vector with 2M components. In this linear case, the minimum of the cost function has a unique solution when rank (**A**) = 8. It is given by the "pseudo-inverse" of the **A** matrix: $\mathbf{u} = (\mathbf{A}^t.\mathbf{A})^{-1}.\mathbf{b}$. It turns out that the pattern shift method reduces to a linear system. Moreover, in this case, the eigen values of the system can be used to analyse the sensitivity of the solution to the experimental parameters.

3. « 3D Hough » method

The 3D Hough method [4] is based on the fact that EBSD diagrams result from electron diffraction, so that the lines observed are in fact hyperboles due to the intersection between a diffracting cone and a plane. Figure 2 gives a simulated EBSD diagram, together with its 3D Hough transform obtained by the following formula:

$$I(\rho,\theta,\alpha) = \frac{1}{N}\sum_{(x,y)\in H_{\rho,\theta,\alpha}} I(x,y) \quad (2)$$

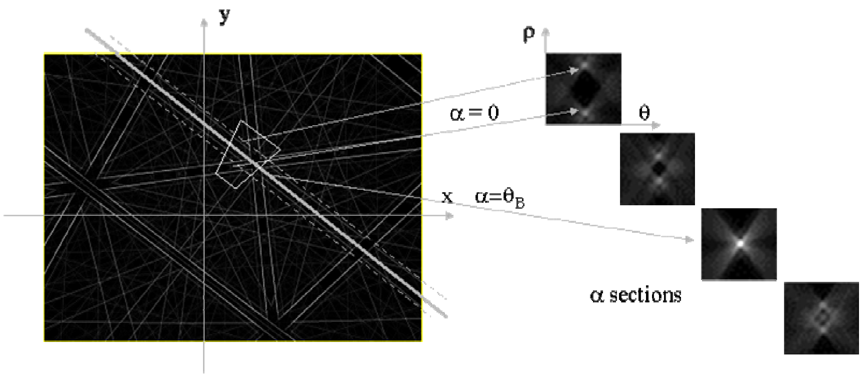

Figure 2 – Local 3D Hough transform applied to a simulated EBSD diagram

In equation (2), $H_{\rho,\theta,\alpha}$ is the hyperbole characterised by the diffracting plane, which produces the straight line (ρ,θ) on the diagram, and by a diffraction angle α, which produces for example the two dashed hyperboles on the diagram. For α=0, the upper right α section in figure 2 is identical to a classical Hough transform applied to the part in the diagram inside the depicted trapeze. This produces two diffuse points associated with the two hyperboles. When α reaches the diffracting angle θ_B, then the (x,y) points associated with the hyperboles are located on the straight line of the diffracting plane. As a consequence, when this point is localised, then the diffracting plane associated with the hyperboles is clearly identified.

The main advantages of the 2D Hough transform are the following:

- The EBSD diagram can be analysed with a very large accuracy in term of orientation
- The distortions of the EBSD diagram due to an elastic strain (the deviatoric part) can be directly detected on the diagram, i.e. without any reference. Using this method, a polycrystalline material can thus be analysed.
- Other types of diagrams can be analysed, as for example Kossel diffraction diagrams for which the curvature of the hyperboles are much more pronounced.

4. Références

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